ENERGY EXCHANGE ISTANBUL

DAY AHEAD ELECTRICITY MARKET

MARKET CLEARING ALGORITHM

2016

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1. INTRODUCTION

In this document, we introduce a new software to solve day-ahead electricity market clearing problem in Turkish electricity system. The algorithm has been developed in the scope of a new domestic software development project for the operation of Turkish day-ahead electricity market. The former market management system (e-terra), which had been in use between December 1st 2011 and May 31st, 2016, was replaced by the new software. The project was initiated in August 2015 and the new software has been in use since June 1st, 2016.

The optimization engine of the software requires market data consisting of bids from the market participants and returns hourly market clearing prices (MCPs) and matching prices and quantities for each bid. The engine includes several problem specific algorithms and a mathematical model is used to solve the problem to optimality. Since the problem has to be solved in a restricted time, the engine guarantees to find a feasible solution first and then tries to find the optimal solution to the problem.

In the second section, we briefly present the different types of bids available in the Turkish market. In Section 3, we give the mathematical representation of the problem, which is determined by the rules in [1], [2] and [3]. We present the overall solution method in Section 4 and conclude the report in Section 5.

2. BID TYPES

In this section, we present the bid types available in the Turkish electricity day-ahead market (EDAM) and describe their features. The detailed features of the bids are covered in Article 53 in [1].

There are three different types of bids: Single (hourly), block and flexible. Each bid consists of quantity-price pairs (QPPs). Each QPP represents either a supply or demand of energy. By convention, supply and demand quantities are given in negative and positive values, respectively. Let q denote the quantity and p denote the price of a QPP. In any bid, the pair *(p, q)* shows that the participant is willing to pay at most p per *MWh* to buy q units of electricity or willing to receive at least p per *MWh* to sell q unit of electricity. The unit of quantity is *"lot"* and *1 lot* equals *0.1 MWh.*

***Hourly bids*:** In this bid type, bidders simply specify a list of QPPs to buy or sell electricity for a single hour of the next day. Detailed features of the hourly bids are covered in Article 5 in [2].

Prices in an hourly bid must be stated as an increasing sequences. Since bidders are expected to buy electricity at lower prices and sell at higher prices, corresponding quantities in the hourly bid must be a non-increasing sequences. Tables 2.1 – 2.3 show hourly bid examples.

Table 2.1 shows an hourly bid of a participant, who buys 100 lot whatever the MCP is at that period. This kind of bids are called *price-independent* bids.

Table 2.1. Hourly bid of participant 1

Price (₺/MWh)	0	2000
Quantity (lot)	100	100

According to rules stated in [1], the quantity submitted in minimum or maximum price limit might be partially accepted in case the MCP hits the minimum or maximum price limit, respectively (curtailment case). To illustrate, the matching quantity for the hourly bid in Table 2.1 can be less than 100 lot only if the corresponding MCP is 2000 ₺/MWh.

In the second example, there are 6 price levels in the bid. The participant is willing to buy electricity as long as the unit price is lower than 250 ₺ and sell electricity when the unit price is higher.

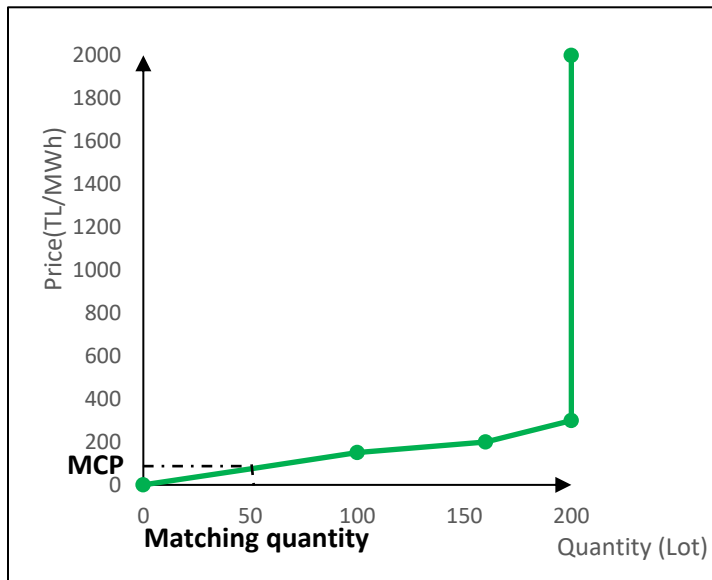**Table 2.2.** Hourly bid of participant 2

Price (₺/MWh)	0	120	200	250	300	2000
Quantity (lot)	100	100	50	0	-50	-100

As another example, the bid shown in Table 2.3 has 5 price levels with different supply quantities.

Table 2.3. Hourly bid of participant 3

Price (₺/MWh)	0	150	200	300	2000
Quantity (lot)	0	-100	-160	-200	-200

Figure 2.1. Piecewise linear supply curve of participant 3

Although an hourly bid consists of a discrete set of quantity price pairs, it is in fact a piecewise linear function generated by linearly interpolating those pairs. The matching quantity of an hourly bid is the quantity corresponding to market clearing price on the piecewise linear function associated with the bid. Figure 2.1 shows supply curve of participant 3. In this case, if MCP is 75₺/MWh, then the matching quantity is 50 lot for this participant.

Block bids: These bids are the second most commonly used type of bid in the Turkish EDAM, and a valid block bid must follow the rules in Article 6 in [2] and Articles 4 and 5 in [3]. Block bids can be viewed as an indivisible set of consecutive hourly bids. However, there is only one QPP in a block bid which is constant for all the periods that the block bid spans. In this case, in addition to price and quantity, bidders also specify the number of consecutive set of hours during which they are willing to trade electricity in the next day. A block bid quantity is either totally accepted or totally rejected for all the periods it spans.

Block bids can be linked to each other in order to depend the acceptance of a block bid to a set of other block bids. When a block bid (*child*) is linked to another block bid (*parent*), child block bid cannot be accepted unless its parent is accepted. At most six block bids can be linked to each other in such a way that there are at most three generations and at most three children at each generation (For more information, see [4]). Linked block bids cannot form a loop, i.e., if block bid A is linked to B, then B cannot linked to A. Given a supply block bid, the linked block bid cannot be a demand block bid, or vice versa. Figure 2.2 shows three possible linked block bid families allowed in Turkish EDAM.

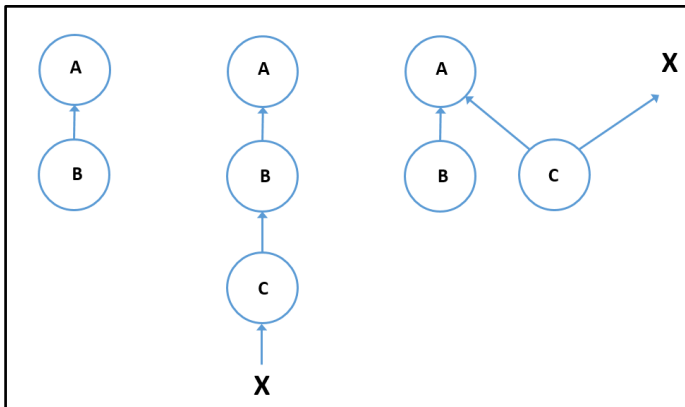**Figure 2.2.** Set of linked block bid configurations in Turkish EDAM.

Table 2.4 shows two block bids, where block bid B is the child bid of block bid A. If B is to be accepted, then A must be accepted as well.

Table 2.4. Linked block bid example

Block Bid	Period	Price(₺)	Quantity(lot)	Link
A	1-5	150	-150	-
B	3-9	10	-100	A

Flexible bids: Flexible bids specify only a price and quantity information and their structures are specified in Article 7 in [2] and Article 6 in [3]. A flexible bid can be accepted at any single hour of the day. Only supply flexible bids can be submitted in the current Turkish EDAM. A flexible bid is either fully accepted or completely rejected. A flexible bid is accepted

if its price is less than or equal to the maximum MCP in the trading day. However, the period accepted does not have to coincide with the period of maximum MCP.

3. PROBLEM DESCRIPTION

3.1. Notation

Sets and Indices:

- ➢ t, T: time period and set of time periods
- ➢ I: set of hourly supply bids
- ➢ J: set of hourly demand bids
- ➢ l, L(i): segment index and set of segments for hourly bid i, $i \in I \cup J$
- ➢ B^s: set of supply block bids (B^{sc}: set of child supply block bids, $B^{sc} \subset B^s$)
- ➢ B^d: set of demand block bids (B^{dc}: set of child demand block bids, $B^{dc} \subset B^d$)
- ➢ Λ^b: set of block bids to which block bid b is linked, $b \in B^s \cup B^d$ (All block bids in Λ^b must be accepted to accept block bid b.)
- ➢ F^s: set of supply flexible bids

Parameters:

- ➢ P_{min}: the minimum valid bid price
- ➢ P_{max}: the maximum valid bid price
- ➢ P_{itl}^0, P_{itl}^1: the initial and final price of segment l of hourly bid i in period t ($P_{min} \leq P_{itl}^0 < P_{itl}^1 \leq P_{max}$ for supply bids and $P_{max} \geq P_{itl}^0 > P_{itl}^1 \geq P_{min}$ for demand bids)
- ➢ Q_{itl}^0, Q_{itl}^1: the initial and final quantity of the segment l of hourly bid i in period t ($0 \leq Q_{itl}^0 \leq Q_{itl}^1$ for all bids)
- ➢ P_b, P_f: the price for block bid b and flexible bid f
- ➢ Q_b, Q_f: the quantity for block bid b and flexible bid f
- ➢ N_b: the number of time periods spanned by block bid b, $b \in B^s \cup B^d$
- ➢ δ_{bt}: a binary parameter equal to 1 if block bid b spans period t, 0 otherwise.

Decision Variables:

- ➢ p_t: MCP at period t
- ➢ x_{itl}: acceptance ratio of segment l of hourly bid i in period t
- ➢ y_b: 1 if block bid b is accepted, 0 otherwise

➢ z_{ft}: 1 if flexible bid f is accepted in period t, 0 otherwise

3.2.   Formulation

In day-ahead electricity markets, participants submit different types of bids for any number of periods of the next day. The market operator has to decide on the accepted quantities for each bid or, in other words, *matching quantities*. Matching rules are indicated by Articles 9 to 11 in [2]. The market operator determines matching quantities for each bid in such a way that daily market surplus is maximized while total supply and demand are balanced at each period. In addition, each type of bid has different constraints on the accepted quantities that must be satisfied by the resulting MCPs.

Each hourly bid is a piecewise linear function formed with the given QPPs and the linear interpolation principle. This function returns the bid quantity for each price value. Hence, matching quantity of an hourly bid is the quantity returned by this function at the MCP.

A supply block bid is accepted if the bid price is less than or equal to average MCP of the periods spanned by the block bid. Likewise, a demand block bid is accepted if the bid price is greater than or equal to average MCP of the periods the bid spans. This constraint is not enforced on child block bids. A supply flexible bid is accepted if the bid price is less than or equal to the maximum MCP. Since the problem has a non-convex structure, there may exist some block or flexible bids which are accepted although the above conditions do not hold. We call these bids as *paradoxically accepted bids (PABs)*. Since, PABs generates negative surplus for the bidder, side payment is paid to the bidder to compensate the associated economical loss.

The objective function of the problem is the maximization of the daily market surplus. The daily market surplus is defined as the sum of daily surpluses of producers and consumers resulting from their trading activities in the day-ahead market. The daily *producer surplus* of a participant is the total revenue earned by selling the matching quantity to the market minus the total amount of money required by the participant to sell that much quantity to the market. The daily *consumer surplus* of a participant is the total amount of money that the participant is willing to pay to purchase the matching quantity minus the amount paid for this quantity.

Market surplus is the sum of the surpluses generated by the accepted hourly, block and flexible bid quantities. Figure 3.2.1 demonstrates the total consumer and producer surplus of matched hourly bids for a given period. The associated supply and demand curves are formed by aggregating the individual supply and demand hourly bids submitted for the given period, respectively. The consumer surplus corresponds to the area defined by the demand curve, P-Q axes and the MCP value (blue polygon) while the producer surplus is the area defined by

the supply curve, P-Q axes and the MCP value (green polygon). Note that, due to accepted block and flexible bids, the MCP may not form at the point where aggregated supply and demand curves of hourly bids intersect.

Figure 3.2.1. Consumer and producer surpluses associated with aggregate hourly bids
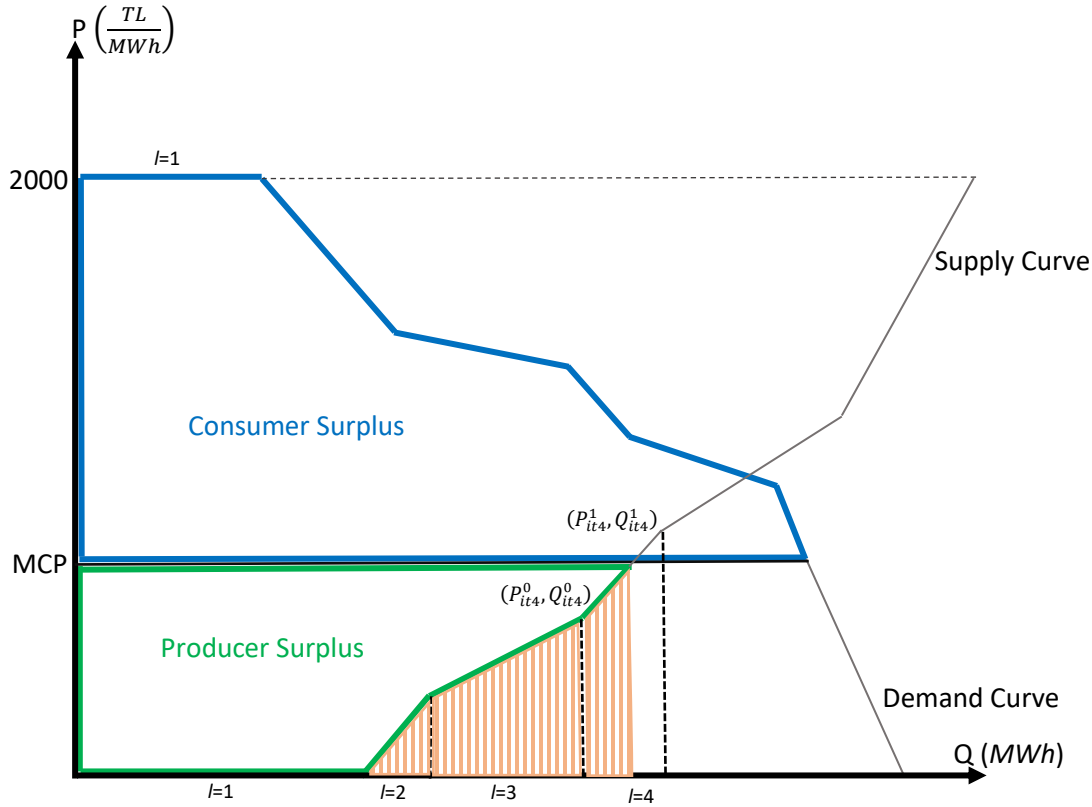
We represent an hourly bid as a set of segments in our mathematical model. A segment is the area between two consecutive QPPs of supply and demand curves of an hourly bid. If the supply quantity at the minimum price is greater than zero, we form an additional segment and add it between this quantity and the price axis. Similarly, if the demand quantity at the maximum price is greater than zero, then we add such an additional segment between this quantity and the price axis.

Producer surplus generated by hourly bids:

The first term shows the total revenue earned by selling the matching quantity to the market, whereas the second term corresponds to the total amount of money required by the participant to sell that much quantity to the market. The latter is shown by the shaded area in Figure 3.2.1.

$$\left(\sum_{t \in T} p_t \sum_{i \in I} \sum_{l \in L(i)} (Q_{itl}^1 - Q_{itl}^0) * x_{itl} \right) - \left(\sum_{t \in T} \sum_{i \in I} \sum_{l \in L(i)} [0.5 x_{itl}(Q_{itl}^1 - Q_{itl}^0)(2P_{itl}^0 + x_{itl}(P_{itl}^1 - P_{itl}^0))] \right)$$

Consumer surplus generated by hourly bids:

$$\left(\sum_{t \in T} \sum_{i \in J} \sum_{l \in L(i)} [0.5 x_{itl}(Q_{itl}^1 - Q_{itl}^0)(2P_{itl}^0 + x_{itl}(P_{itl}^1 - P_{itl}^0))] \right) - \left(\sum_{t \in T} p_t \sum_{i \in J} \sum_{l \in L(i)} x_{itl} (Q_{itl}^1 - Q_{itl}^0) \right)$$

Figure 3.2.2 shows the producer and consumer surpluses associated with an accepted supply bid and an accepted demand block bid, respectively. Let P* be the average MCP of the periods where the block bid is active. Then, the surplus of an accepted block bid equals to the volume of the illustrated rectangular prism, i.e. the difference between P* and price of the block bid multiplied by the number of periods (N_b) and absolute matching quantity of the block bid (q_b^*).

Figure 3.2.2. Illustration of surpluses for accepted supply and demand block bids
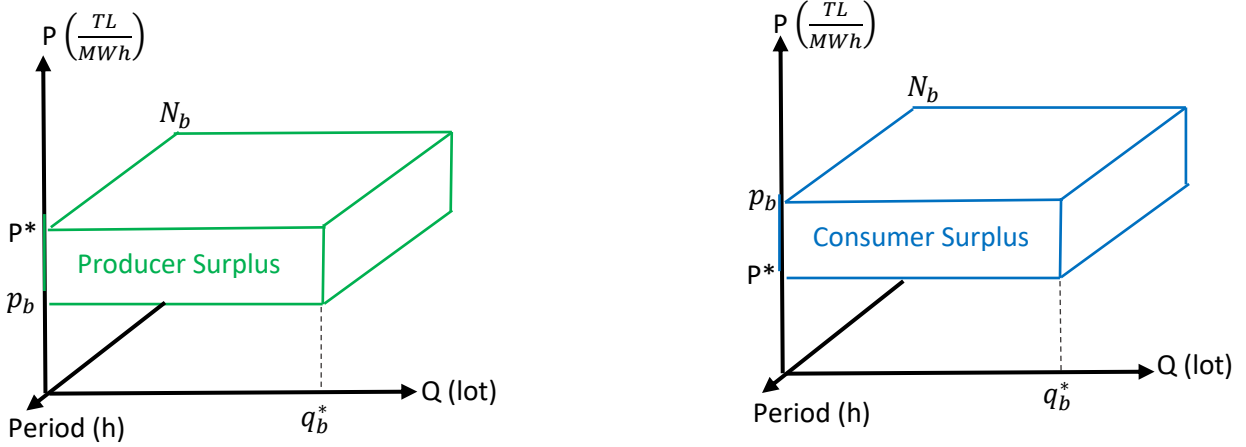
Producer surplus generated by block bids

$$\sum_{b \in B^s} y_b * Q_b * \left(\sum_{t \in T} \delta_{bt} p_t - N_b P_b \right)$$

Consumer surplus generated by block bids

$$\sum_{b \in B^d} y_b * Q_b * \left(N_b P_b - \sum_{t \in T} \delta_{bt} p_t \right)$$

The surplus of an accepted supply flexible bid is calculated in a similar way to the surplus of an accepted supply block bid with $N_b = 1$.

Producer surplus generated by flexible bids

$$\sum_{f \in F^s} Q_f * \left(\sum_{t \in T} z_{ft} \, p_t - P_f \right)$$

A feasible market result must satisfy the following constraints:

a. At every period, accepted demand and supply quantities must be equal.

$$\sum_{i \in I} \sum_{l \in L(i)} (Q_{itl}^1 - Q_{itl}^0) \, x_{itl} + \sum_{b \in B^s} \delta_{bt} Q_b y_b + \sum_{f \in F^s} Q_f z_{ft}$$
$$- \sum_{i \in J} \sum_{l \in L(i)} (Q_{itl}^1 - Q_{itl}^0) \, x_{itl} - \sum_{b \in B^d} \delta_{bt} Q_b y_b = 0, \quad \forall t \in T$$

b. The matching quantity of an hourly bid must be equal to the offered bid quantity corresponding to the market clearing price of the period. If the price set of an hourly bid does not contain the market clearing price, the matching quantity is found by linear interpolation.

For supply segments: $\forall i, t, l$
$$x_{itl} = \begin{cases} 1, & if \ P_{itl}^1 < p_t \\ (p_t - P_{itl}^0)/(P_{itl}^1 - P_{itl}^0), & if \ P_{itl}^0 \le p_t \le P_{itl}^1, l \neq 1 \\ 0, & if \ p_t < P_{itl}^0 \end{cases}$$

For demand segments: $\forall j, t, l$
$$x_{itl} = \begin{cases} 1, & if \ p_t < P_{itl}^1 \\ (p_t - P_{itl}^0)/(P_{itl}^1 - P_{itl}^0), & if \ P_{itl}^1 \le p_t \le P_{itl}^0, l \neq 1 \\ 0, & if \ P_{itl}^0 < p_t \end{cases}$$

c. Block bids are either totally accepted or rejected, partial acceptance is not allowed. If a block bid is accepted, it must be accepted at all periods it covers.

$$y_b \in \{0,1\}, \ \forall b \in B^s \cup B^d$$

d. Flexible bids are either totally accepted or rejected, partial acceptance is not allowed.

$$z_{ft} \in \{0,1\}, \ \forall f \in F^s, \ t \in T$$

e. Supply block bids having bid price less than or equal to the average MCP of the periods covered by the block bid must be accepted. Similarly, demand block bids having bid price greater than or equal to the average MCP of the periods covered by the block bid must be accepted. These constraints are not enforced for child block bids.

Supply block bids:

$$N_b P_b \leq \sum_{t \in T} \delta_{bt} p_t \Rightarrow y_b = 1, \qquad \forall b \in B^s \backslash B^{sc}$$

Demand block bids:

$$N_b P_b \geq \sum_{t \in T} \delta_{bt} p_t \Rightarrow y_b = 1, \qquad \forall b \in B^d \backslash B^{dc}$$

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f. A child block bid cannot be accepted unless its parent bid is accepted as well.

$$y_b = 1 \Rightarrow y_\lambda = 1, \ \forall \lambda \in \Lambda^b, \qquad b \in B^{sc} \cup B^{dc}$$

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g. A supply flexible bid must be accepted if maximum MCP is greater than or equal to the bid price.

$$P_f \leq \max_{t \in T} p_t \Rightarrow \sum_{t \in T} z_{ft} = 1, \qquad \forall f \in F^s$$

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h. A flexible bid can be accepted at most one period.

$$z_{f\hat{t}} = 1 \Rightarrow z_{ft} = 0 \ \forall t \in T \backslash \{\hat{t}\}, \forall f \in F^S, \forall \hat{t} \in T$$

Although these constraints must be satisfied in the announced market result, a feasible solution cannot be guaranteed in some extreme market situations. In case of supply deficit or surplus situations, the constraints *b, e* and *g* are relaxed to guarantee a feasible solution for the market.

We model this problem as a *Mixed-Integer Quadratic Program* (MIQP) and try to find the optimal solution of the problem. The obtained solution includes the MCP for each period and hourly accepted quantities for each bid. Although the solution values are stored as double-precision floating-point numbers in the algorithm, the announced MCPs and accepted quantities (in lots) are rounded to values with two and zero decimal digits, respectively.

3.3. Supply Demand Imbalance and Curtailment Procedure

The problem does not necessarily have a feasible solution so that there may not exist an MCP between the price limits for a period to balance total supply and demand under the constraints defined in a-h. In such instances, energy surplus or energy deficit situation occurs. In order to keep supply and demand in balance in such a situation, hourly bids are curtailed and acceptance constraints of block and flexible bids are relaxed.

Energy surplus occurs when the total supply at the lower price limit, $Supply\ (P_{min})$, is higher than the total demand at this price, $Demand\ (P_{min})$. If this situation occurs for a period, the MCP will be equal to the lower price limit for that period. The amount of energy surplus is the difference between $Supply\ (P_{min})$ and $Demand\ (P_{min})$. The matching quantity of an hourly supply bid i at time t, q_{it}^*, is calculated as follows when energy surplus occurs at period t.

$$Curtailed\ quantity\ =\ Supply\ (P_{min}) - Demand\ (P_{min})$$
$$Curtailment\ ratio\ =\ Curtailed\ quantity\ /\ Supply\ (P_{min})$$
$$q_{it}^* = (Q_{it1}^1 - Q_{it1}^0) \times Curtailment\ ratio,\ i \in I$$

Energy deficit occurs when the total supply at the upper price limit, $Supply\ (P_{min})$, is less than the total demand at this price, $Demand\ (P_{max})$. If this situation occurs for a period, the MCP will be equal to the upper price limit for that period. The amount of energy deficit is the difference between $Demand\ (P_{max})$ and $Supply\ (P_{max})$. The matching quantity of an hourly demand bid i at time t, q_{it}^*, is calculated as follows when energy deficit occurs at period t.

$$Curtailed\ quantity\ =\ Demand\ (P_{max}) - Supply\ (P_{max})$$
$$Curtailment\ ratio\ =\ Curtailed\ quantity\ /\ Demand\ (P_{max})$$
$$q_{it}^* = (Q_{it1}^1 - Q_{it1}^0) \times Curtailment\ ratio,\ i \in J$$

4. SOLUTION METHOD

In this section, we describe the process to solve the problem defined in the previous section. The process consists of four main modules: pre-processing, heuristic methods, optimization and post-processing. In each module, there are several algorithms specifically developed for this problem. In addition to these modules, there are verification and repair algorithms used to guarantee a valid market result in all market circumstances. The first three modules are run successively whereas the algorithms in post-processing module excluding the reporting part can be used at different points of the implementation. Reporting is the final step of the solver in which the solution is converted to the market result format.

Figure 4.1. Flow chart of the solution method
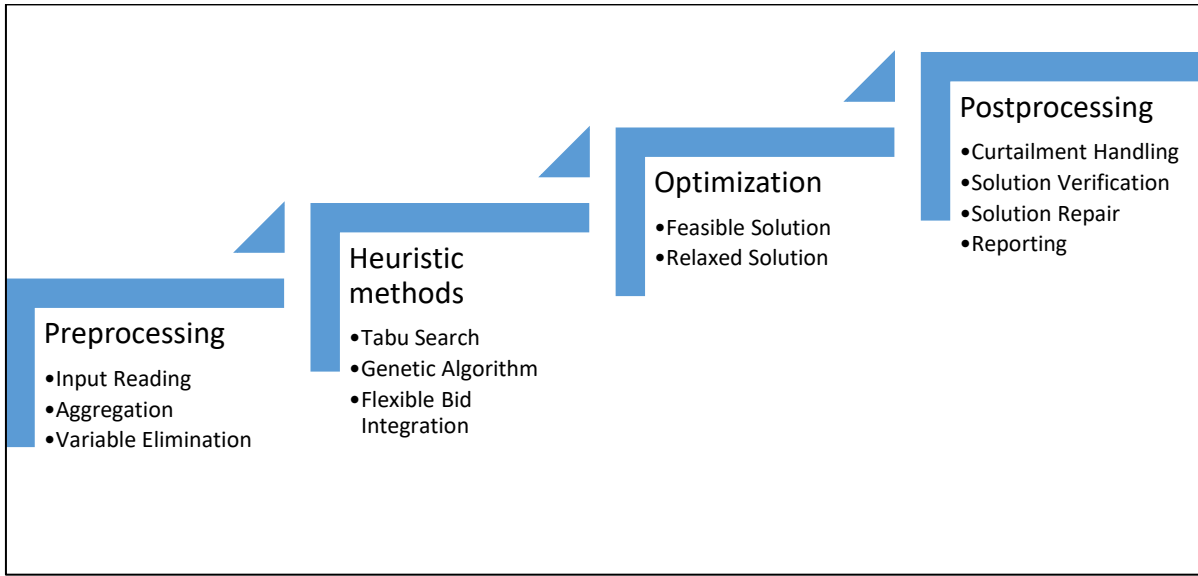
1. *Preprocessing:* In this module, aggregation is the procedure of converting individual hourly bids to a single hourly supply and demand bid for each period. The goal of the variable elimination algorithm is to reduce the problem size without sacrificing the optimal solution.

2. *Heuristic methods*: We run two heuristic algorithms in parallel to find a good starting solution for the optimization module. Heuristic methods try to find best block bid decisions excluding the flexible bids. Flexible bid decisions are given afterwards in the flexible bid integration algorithm.
The solutions generated by the heuristics are compared and the better one is selected as the initial solution of the optimization module. This solution is the final solution in case optimization module cannot find a better solution in the given time limit.

3. *Optimization:* Given the initial solution, a commercial mathematical programming solver is called to find the optimal solution. We both solve the original problem and its relaxation (relaxing constraints *e* and *g* in Section 3). If the optimal solution of the relaxed problem is feasible for the original problem, it is the final solution. If not, we solve the original problem, repair the optimal solution of the relaxed problem and compare the repaired solution with the best solution found for the original problem. The better one in terms of total market surplus is determined as the final solution.

4. *Post-processing:* This module includes curtailment handling procedure, solution verification and repair algorithms and report generation procedure. We use the solution verification algorithms to check the validity of the solutions found by heuristic methods and the optimization. In case of invalid solutions, we run repair algorithms to

fix them. Curtailment handling is the procedure when there is supply surplus or deficit in the market. In these situations, hourly supply and demand is balanced by relaxing some of the constraints in Section 3 (*b, e and g*).

5. CONCLUSION

In this document, we introduced the market clearing problem in Turkish electricity day-ahead market, provided the problem formulation (objective function and constraints) and presented the implementation details in a comprehensive manner. The problem formulation is constructed in line with the market rules specified in [1], [2] and [3]. In the new formulation, block and flexible bids are guaranteed to be accepted when the specified conditions hold between the relevant MCPs and bid price. The objective function is the maximization of the total surplus which is a necessary condition for an effective operation of the market. We use sophisticated algorithms and a state-of-the art solver to find the optimal solution of the problem.

Market clearing of day ahead markets contains various research problems and improvement opportunities. The foundation of EXIST together with a research and development team dedicated to this project has paved the way for the development of this software with the company's own resources. The completion of this project was the initial step towards a more developed day ahead electricity market in Turkey. With our ongoing research on this area, our agenda includes the assessment of different applications in well-developed markets and the suggestions of the market participants as long as they provide an enhancement to the current market design.

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